

Prof. Spera

FACOLTÀ DI SCIENZE MM.FF.NN.
Corso di laurea in Scienze Biologiche

Prova scritta di Matematica
del 28-04-2010

Cognome..... Nome..... N° Matricola.....

1. Calcolare la derivata delle seguenti funzioni:

$$f(x) = 7x + e^{7x} \cos(5x) + 2e^{7x} \sin(5x);$$

$$g(x) = -4x + 2x^{5 \cos x} + 1.$$

2. Calcolare il seguente limite:

$$\lim_{x \rightarrow +\infty} \left[\frac{\sqrt{3x^4 - 2} - \sqrt{3x^4 + 2}}{5} + 2 \right].$$

3. Enunciare, dimostrare e commentare il teorema di unicità del limite.

4. Studiare la seguente funzione:

$$f(x) = \frac{4}{5} - \frac{x^2 - 4}{x^2 - 1}.$$

5. Calcolare il seguente integrale indefinito:

$$\int \frac{2}{3} x^3 (\lg x)^2 dx. \quad \text{Svelto}$$

$$f(x) = 7x + e^{7x} \cos(5x) + 2 e^{7x} \sin(5x) \Rightarrow$$

$$f'(x) = 7 + \underline{7e^{7x} \cos 5x} - \underline{5e^{7x} \sin 5x} + \underline{14e^{7x} \sin 5x} + \underline{10e^{7x} \cos(5x)} =$$

$$= 7 + 17 \cos(5x) + 9 e^{7x} \sin 5x$$

$$g(x) = -4x + 2x^{5\cos x} + 1 \Rightarrow \cancel{-4x + 2(x^{5\cos x})'}$$

$$g'(x) = -4 + 2 \underbrace{(x^{5\cos x})'}_{(A)}$$

$$(A) \quad y = x^{5\cos x}$$

$$\log y = \log(x^{5\cos x}) \Rightarrow$$

$$\log y = 5\cos x \cdot \log x \Rightarrow$$

$$(\log y)' = (5\cos x \cdot \log x)' \Rightarrow$$

$$\frac{y'}{y} = 5 \left(-\sin x \log x + \frac{\cos x}{x} \right)$$

$$y' = \left(-5 \sin x \log x + \frac{5 \cos x}{x} \right) \cdot y$$

$$y' = \left(-5 \sin x \cdot \log x + \frac{5 \cos x}{x} \right) \cdot x^{5\cos x}$$

$$\Rightarrow g'(x) = -4 + 2 \left(-5 \sin x \log x + \frac{5 \cos x}{x} \right) \cdot x^{5\cos x}$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{3x^4-2} - \sqrt{3x^4+2}}{5} + 2 =$$

$$\stackrel{1a}{=} \lim_{x \rightarrow +\infty} \frac{\sqrt{3x^4-2} - \sqrt{3x^4+2}}{5} + 2$$

Calcolo $\lim_{x \rightarrow +\infty} \sqrt{3x^4-2} - \sqrt{3x^4+2} = \infty - \infty$ forma indeterminata

RAZIONABILIZZARE

$$\lim_{x \rightarrow +\infty} \left(\sqrt{3x^4-2} - \sqrt{3x^4+2} \right) \cdot \frac{\left(\sqrt{3x^4-2} + \sqrt{3x^4+2} \right)}{\left(\sqrt{3x^4-2} + \sqrt{3x^4+2} \right)} \stackrel{\text{formula}}{=} \lim_{x \rightarrow +\infty} \frac{(3x^4-2) - (3x^4+2)}{\sqrt{3x^4-2} + \sqrt{3x^4+2}} =$$

$(a-b)(a+b) = a^2 - b^2$

$$= \lim_{x \rightarrow +\infty} \frac{3x^4-2-3x^4-2}{\sqrt{3x^4-2} + \sqrt{3x^4+2}} = \lim_{x \rightarrow +\infty} \frac{-4}{\sqrt{3x^4-2} + \sqrt{3x^4+2}} = \lim_{x \rightarrow +\infty} \frac{-4}{\infty + \infty} =$$

$$= \frac{-4}{\infty} = 0$$

Ora $\lim_{x \rightarrow +\infty} \frac{\sqrt{3x^4-2} - \sqrt{3x^4+2}}{5} + 2 = \frac{0}{5} + 2 = \boxed{2}$

VERIFICATO

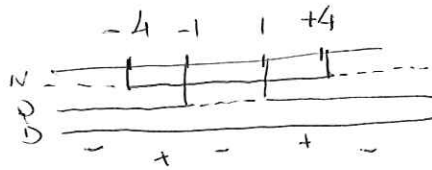
$$f(x) = \frac{4}{5} - \frac{x^2-4}{x^2-1}$$

(D) $x^2-1 \neq 0 \Rightarrow x \neq \pm 1$
 $]-\infty; -1[\cup]-1; 1[\cup]1; +\infty[$

(C) $\frac{4}{5} - \frac{x^2-4}{x^2-1} \geq 0 \Rightarrow \frac{4(x^2-1) - 5(x^2-4)}{5(x^2-1)} = \frac{4x^2-4-5x^2+20}{5(x^2-1)} = \frac{-x^2+16}{5(x^2-1)} \geq 0$

$\begin{cases} -x^2+16 \geq 0 \\ 5 > 0 \\ x^2-1 > 0 \end{cases}$

$$\frac{4}{5} - \frac{16-4}{16-1} = \frac{4}{5} - \frac{12}{15} = \frac{4}{5} - \frac{4}{5} = 0$$



(L)

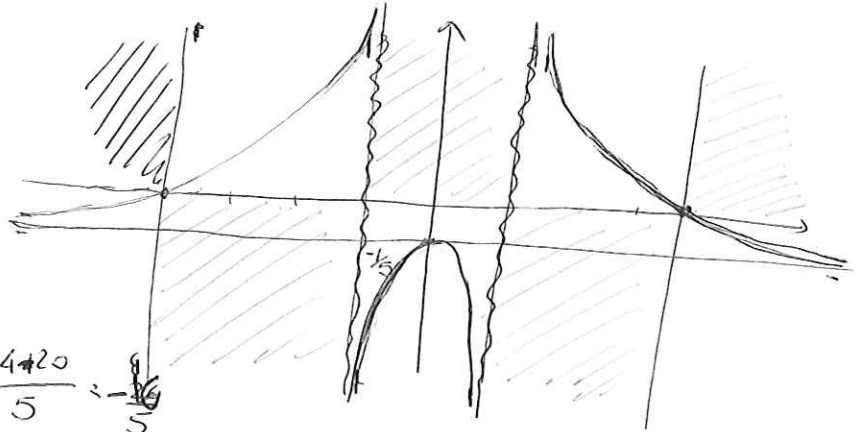
$$\lim_{x \rightarrow \pm\infty} \frac{4}{5} - \frac{x^2-4}{x^2-1} = \frac{4}{5} - 1 = \frac{4-5}{5} = -\frac{1}{5}$$

$$\lim_{x \rightarrow \pm\infty} f(x) = \pm\infty$$

$$f(\pm 4) = 0$$

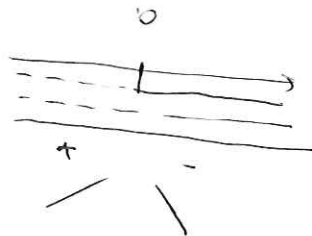
INT ASSEY $\Rightarrow x=0 \Rightarrow y = \frac{4}{5} + 4 = \frac{4+20}{5} = \frac{24}{5}$

$P(0; -\frac{16}{5})$ int. wry



$$(y') = - \frac{2x(x^2-1) - (x^2-4)2x}{(x^2-1)^2} = - \frac{2x(x^2-1 - (x^2-4))}{(x^2-1)^2} = - \frac{2x(x^2-1-x^2+4)}{(x^2-1)^2} = - \frac{2x(3-x^2)}{(x^2-1)^2}$$

$$= \frac{-6x}{(x^2-1)^2} \geq 0$$

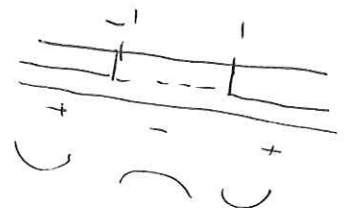


max $(0; -\frac{16}{5})$
 rel.

$$y'' = \frac{-6(x^2-1)^2 + 6x \cdot 2(x^2-1) \cdot 2x}{(x^2-1)^4} = \frac{(x^2-1)[-6(x^2-1) + 24x^2]}{(x^2-1)^4} =$$

$$= \frac{-6x^2+6+24x^2}{(x^2-1)^3} = \frac{18x^2+6}{(x^2-1)^3} \geq 0$$

$\begin{cases} 18x^2+6 \geq 0 \quad \forall x \\ x^2-1 > 0 \end{cases}$



200m

$$\int \frac{2}{3} x^3 \lg^2 x \, dx = \frac{2}{3} \int x^3 \lg^2 x \, dx = \frac{2}{3} \left[\frac{x^4}{4} \lg^2 x - \int \frac{x^4}{4} \cdot \frac{2 \lg x}{x} \, dx \right] =$$

$$= \frac{2}{3} \frac{x^4}{4} \lg^2 x - \frac{2}{3} \int \frac{x^4}{4} \frac{2 \lg x}{x} \, dx = \frac{x^4 \lg^2 x}{6} - \frac{1}{3} \int x^3 \lg x \, dx =$$

$$= \frac{x^4 \lg^2 x}{6} - \frac{1}{3} \left[\frac{x^4}{4} \lg x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx \right] =$$

$$= \frac{x^4 \lg^2 x}{6} - \frac{1}{12} x^4 \lg x + \frac{1}{12} \int x^3 \, dx =$$

$$= \frac{x^4 \lg^2 x}{6} - \frac{x^4 \lg x}{12} + \frac{1}{12} \cdot \frac{x^4}{4} = \frac{x^4 \lg^2 x}{6} - \frac{x^4 \lg x}{12} + \frac{x^4}{48} =$$

$$= \frac{x^4}{6} \left(\lg^2 x - \frac{\lg x}{2} + \frac{1}{8} \right) + C$$

VERIFICA

$$\frac{2}{3} x^3 \left(\lg^2 x - \frac{\lg x}{2} + \frac{1}{8} \right) + \frac{x^4}{6} \left(\frac{2 \lg x}{x} - \frac{1}{2x} \right) =$$

$$= \cancel{\frac{2}{3}} x^3 \lg^2 x - \cancel{\frac{2}{3}} \cdot \frac{1}{2} x^3 \lg x + \frac{2}{3} x^3 \cdot \frac{1}{8} + \frac{x^4}{6} \cdot \frac{2 \lg x}{x} - \frac{x^4}{6} \cdot \frac{1}{2x}$$

$$= \frac{2}{3} x^3 \lg^2 x - \frac{1}{3} x^3 \lg x + \frac{x^3}{12} + \frac{x^3 \lg x}{3} - \frac{x^3}{12} \quad \checkmark$$